FUZZY DATA RECALL USING POLYNOMIAL BIDIRECTIONAL HETERO-CORRELATOR

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ABSTRACT

A method of fuzzy data recall using polynomial bidirectional hetero-correlator is presented. This has a higher capacity for pattern pair storage than that of the conventional BAMs and fuzzy memories. A new energy function is defined. The polynomial bidirectional hetero-correlator (**PBHC**) takes advantage of fuzzy characteristics in evolution equations such that the signal-noise-ratio (SNR) is significantly increased. In this work, we prove the stability of fuzzy data recall using polynomial bidirectional hetero-correlator. The increase of SNR consequently enhances the capacity of the polynomial bidirectional hetero-correlator. The capacity of the fuzzy data recall using PBHC in the worst case is also estimated.

KEYWORDS: polynomial bidirectional hetero-correlator (PBHC), SNR, storage capacity

1. INTRODUCTION

Associative memories have been an important research area in neural networks [1], [2], [3], [4], [5], [6], [9], [10]. Kosko presented a fuzzy associative memory (FAM) system structure [7], [8]. However, no energy function introduced in his papers could guarantee that every stored pattern pair resides at a local minimum on energy surfaces. Moreover, there was no capacity analysis given in the papers.

The rest of this paper is organized as follows. Section 2 analyzes the framework of the high capacity polynomial bidirectional hetero-correlator (**PBHC**) in which the component of a fuzzy vector is called a fuzzy bit (**fit**). We present the new evolution equations in the recall process of PBHC in Section 2.1. Next, we propose our energy function and two-phase approach to verify the stability of fuzzy data recall using the PBHC in Section 2.2 which attempts to overcome the deficiency of previous investigations. Herein we also adopt the signal-noise-ratio (SNR) approach and we present the equation of sufficient condition of the polynomial bidirectional hetero-correlator to analyze the Z value, which is deemed as the power of the polynomial, and capacity of the PBHC in Section 2.3. The lower bound solution of Z value attempts to develop a means for the mathematical theory associated with PBHCs to derive the smallest Z value, which can still recall all of the stored pattern pairs such that the dimension of the patterns can be as large as possible. Any Z value, which satisfies the condition of absolute lower bound of the Z value, can recall all of the different patterns stored in the PBHC. In Section 3, the simulation results are given, which outperform those of previous works. Concluding remarks are finally made in Section 4.

2. FRAMEWORK OF HIGH CAPACITY PBHC

2.1. Evolution Equations

Suppose we are given M pattern pairs, which are

$$\{(X_1,Y_1),(X_2,Y_2),\ldots,(X_M,Y_M)\}, \tag{1}$$

where $X_i = (x_{i1},x_{i2},\ldots,x_{in})$, $Y_i = (y_{i1},y_{i2},\ldots,y_{ip})$. Let $1 \le i \le M$, $x_{ij} \in [0,1]$, $1 \le j \le n$, $y_{ij} \in [0,1]$, $1 \le j \le p$, n and p are the component dimensions of X and Y, and n is assumed to be smaller than or equal to p without any loss of generality. Instead of using Kosko's approach [9], we use the following evolution equations in the recall process of PBHC.

$$y_k = \frac{\sum_{i=1}^{i=M} y_{ik} \cdot \left(\frac{u - \|X_i - X\|^2}{u} \right)^{M^Z}}{\sum_{i=1}^{i=M} \left(\frac{u - \|X_i - X\|^2}{u} \right)^{M^Z}} \tag{2}$$

$$x_k = \frac{\sum_{i=1}^{M} x_{ik} \cdot \left(\frac{u - \|Y_i - Y\|^2}{u} \right)^{M^Z}}{\sum_{i=1}^{M} \left(\frac{u - \|Y_i - Y\|^2}{u} \right)^{M^Z}} \tag{3}$$

where M is the number of patterns in the PBHC, X_i, Y_i, $i = 1,\ldots,M$, are the stored patterns, X *or* Y is the initial vector presented to the network, x_k and x_{ik} are the kth digits of X and X_i, respectively, y_k and y_{ik} are the kth digits of Y and Y_i, respectively, Z is a positive integer, and u is a function defined as the following equation:

$$u = \sum_{i=1}^{M}\sum_{j=1}^{M}\left(\left\|X_i - X_j\right\|^2 + \left\|Y_i - Y_j\right\|^2\right). \quad (4)$$

Note that u is bounded.

2.2. Energy Function and Stability

Since every stored pattern pair should produce a local minimum on the energy surface in order to be recalled correctly, the energy function is intuitively defined as

$$E = \sum_{i=1}^{M} \left\|X - X_i\right\|^2 \cdot \left\|Y - Y_i\right\|^2. \quad (5)$$

The above fuzzy data model using PBHC can be deemed as one kind of BAM, bidirectional associative memory. Therefore, we can explore its stability by studying its two phases of evolution, i.e., $X \to Y$ and $Y \to X$.

Theorem 1 : The PBHC modeled by Eqn.(2) and Eqn.(3) is a stable system.

Proof : We discuss the stability of observing the behavior of energy function of two directions, $X \to Y$ and $Y \to X$, respectively.

Phase 1 : $X \to Y$. We use the energy function as Eqn.(5).
Thus, the $\nabla_{x_k} E(X,Y)$ can be computed as follows,

$$\begin{aligned}\nabla_{x_k} E(X,Y) &= 2\sum_{i=1}^{M}(x_k - x_{ik})\left\|Y - Y_i\right\|^2 \\ &= 2\left(\sum_{i=1}^{M}\left\|Y - Y_i\right\|^2\right) \\ &\quad \cdot \left[x_k - \frac{\sum_{i=1}^{M} x_{ik}\left((u - \left\|Y - Y_i\right\|^2)/u\right)^{M^z}}{\sum_{i=1}^{M}\left((u - \left\|Y - Y_i\right\|^2)/u\right)^{M^z}}\right]. \end{aligned} \quad (6)$$

The difference of E due to a bit's change, therefore, can be derived to be

$$\begin{aligned}\Delta_{x_k} E(X,Y) &= \nabla_{x_k} E(X,Y) \cdot \Delta_{x_k} \\ &= 2\left(\sum_{i=1}^{M}\left\|Y - Y_i\right\|^2\right) \\ &\quad \cdot \left[x_k - \frac{\sum_{i=1}^{M} x_{ik}\left((u - \left\|Y - Y_i\right\|^2)/u\right)^{M^z}}{\sum_{i=1}^{M}\left((u - \left\|Y - Y_i\right\|^2)/u\right)^{M^z}}\right] \\ &\quad \cdot (x_k' - x_k) \\ &= -2\left(\sum_{i=1}^{M}\left\|Y - Y_i\right\|^2\right) \\ &\quad \cdot \left[\frac{\sum_{i=1}^{M} x_{ik}\left((u - \left\|Y - Y_i\right\|^2)/u\right)^{M^z}}{\sum_{i=1}^{M}\left((u - \left\|Y - Y_i\right\|^2)/u\right)^{M^z}} - x_k\right] \\ &\quad \cdot (x_k' - x_k) \end{aligned} \quad (7)$$

According to Eqn.(3), we have the following inequalities when x_k' is the next state of x_k .

Case 1: If

$$x_k - \frac{1}{2\lambda} \le \frac{\sum_{i=1}^{M} x_{ik}\left((u - \left\|Y - Y_i\right\|^2)/u\right)^{M^z}}{\sum_{i=1}^{M}\left((u - \left\|Y - Y_i\right\|^2)/u\right)^{M^z}} \le x_k + \frac{1}{2\lambda}$$

then , $x_k' = x_k$ according to (3). Thus, $\Delta_{x_k} E(X,Y) = 0$.

Case 2: If

$$\frac{\sum_{i=1}^{M} x_{ik}\left((u - \left\|Y - Y_i\right\|^2)/u\right)^{M^z}}{\sum_{i=1}^{M}\left((u - \left\|Y - Y_i\right\|^2)/u\right)^{M^z}} \ge x_k + \frac{1}{2\lambda}$$

then, $x_k' > x_k$. Thus, according to (7), $\Delta_{x_k} E(X,Y) < 0$.

Case 3: If

$$\frac{\sum_{i=1}^{M} x_{ik}\left((u - \left\|Y - Y_i\right\|^2)/u\right)^{M^z}}{\sum_{i=1}^{M}\left((u - \left\|Y - Y_i\right\|^2)/u\right)^{M^z}} < x_k - \frac{1}{2\lambda}$$

then, $x_k' < x_k$. Thus, according to (7), $\Delta_{x_k} E(X,Y) < 0$.
In conclusion, the $X \to Y$ phase causes E to decrease, $\Delta_{x_k} E(X,Y) < 0$.

Phase 2 : $Y \to X$. We again use the energy function as Eqn.(5). Since the procedure of the derivation is very much the same as that of the $X \to Y$ phase, there is no need to repeat the lengthy discussion.

In conclusion, the $Y \to X$ phase also causes E to decrease, Note that the energy function defined in Eqn.(5) is bounded. In short, the $X \to Y$ phase always drags down $E(X,Y)$, while the $Y \to X$ phase also reduce $E(X,Y)$. The evolution will be terminated when $E(X,Y)$ reaches the minimum where the pattern pairs are stored.

2.3. Analysis of Capacity of PBHC

We adopt the SNR approach to compute the capacity of the PBHC. Let X_i and Y_i be the stored pattern pairs. Assume X_1 is the input pattern pair and Y_1 will be recalled expectantly. By substituting X_1 for X, Eqn.(2) can be written as

$$y_k \cdot \sum_{i=1}^{M} \left(\frac{u - \|X_i - X_1\|^2}{u} \right)^{M^z}$$

$$= y_{1k} \cdot 1^2 + y_{2k} \cdot \left(\frac{u - \|X_2 - X_1\|^2}{u} \right)^{M^z}$$

$$+ y_{3k} \cdot \left(\frac{u - \|X_3 - X_1\|^2}{u} \right)^{M^z}$$

$$+ \ldots + y_{Mk} \cdot \left(\frac{u - \|X_M - X_1\|^2}{u} \right)^{M^z} \quad (8)$$

The largest noise that can appear is in the worst case, and which will be just one component different, while the other components stay the same between components X_i and X_1. For instance, $X_1 = (x_{11}, x_{12}, \ldots, x_{1n})$, and $X_i = (x_{11}, x_{12}, \ldots, x_{1n} \pm 1/(2\lambda))$, $i \neq 1$, where $x_{ik}, y_{ik} \in \{0/\lambda, 1/\lambda, \ldots, \lambda/\lambda\}$, fuzzy space = [0,1], λ is fuzzy quantum, and σ is fuzzy quantum gap. Suppose $\lambda = 10$, we can obtain $\sigma = 1/\lambda = 0.1$, and $1/(2\lambda) = \sigma/2 = 0.05$. The first term in the above equation corresponds to the signal, and the other terms are the noise. The power of signal is

$$S = y_{1k} \cdot 1^2. \quad (9)$$

Besides the first term, the rest of the terms are actually the sum of M-1 independent identically distributed random variables. Therefore, the noise of these terms is M-1 times of the noise of a single random variable. From Eqn.(8) we can get the following inequalities:

$$y_k \cdot \sum_{i=1}^{M} \left(\frac{u - \|X_i - X\|^2}{u} \right)^{M^z}$$

$$\leq y_{1k} \cdot 1^2 + \sum_{j=2}^{M} y_{jk} \cdot \left(\frac{u - \frac{1}{\lambda^2}}{u} \right)^{M^z}$$

$$\leq y_{1k} \cdot 1^2 + (M-1) y_{2k} \cdot \left(\frac{u - \frac{1}{\lambda^2}}{u} \right)^{M^z}$$

$$= S + N_{\max}. \quad (10)$$

The first term in the above equation is the signal, and the second term is deemed to be the noise in the worst case.

Let
$y_{2k} = j/\lambda$, $j \in \{0,1,2,3,\ldots,\lambda\}$, $y_{2k} \in \{0/\lambda, 1/\lambda, 2/\lambda, \ldots, \lambda/\lambda\}$.

The sufficient condition for the noise must be bounded is

$$\lambda - \frac{1}{2} < (M-1) \cdot y_{2k} \cdot \left(\frac{u - \frac{1}{\lambda^2}}{u} \right)^{M^z} < \lambda + \frac{1}{2} \quad (11)$$

The above equation can be simplify as follows,

$$\lambda - \frac{1}{2} < (M-1) \cdot \frac{j}{\lambda} \cdot \left(\frac{u - \frac{1}{\lambda^2}}{u} \right)^{M^z} < \lambda + \frac{1}{2}$$

$$-\frac{1}{2} < (M-1) \cdot j \cdot \left(\frac{u - \frac{1}{\lambda^2}}{u} \right)^{M^z} < \frac{1}{2}$$

$$\left| (M-1) \cdot j \cdot \left(\frac{u - \frac{1}{\lambda^2}}{u} \right)^{M^z} \right| < \frac{1}{2}$$

$$\left| (M-1) \cdot \left(\frac{u - \frac{1}{\lambda^2}}{u} \right)^{M^z} \right| < \frac{1}{2j} \leq \frac{1}{2} \quad (12)$$

The worst case will occur when j=1, and we deem the above equation as sufficient condition for the polynomial bidirectional hetero-correlator (PBHC) to recall any pattern correctly.

We next derive the minimal Z in the worst case for the PBHC to correctly recall every stored pattern pair as follows,

$$(M-1) \cdot \left(\frac{u - \frac{1}{\lambda^2}}{u} \right)^{M^z} \leq \frac{1}{2}$$

$$\left(\frac{u - \frac{1}{\lambda^2}}{u} \right)^{M^z} \leq \frac{1}{2(M-1)}$$

$$ln \left(\frac{u - \frac{1}{\lambda^2}}{u} \right)^{M^z} \leq ln \left[\frac{1}{2(M-1)} \right]$$

$$M^z \cdot ln \left(\frac{u - \frac{1}{\lambda^2}}{u} \right) \leq ln \left[\frac{1}{2(M-1)} \right]$$

since

$$ln \left(\frac{u - \frac{1}{\lambda^2}}{u} \right) < 0$$

we obtain

$$M^{Z} \geq \frac{ln\left[\frac{1}{2(M-1)}\right]}{ln\left(\frac{u-\frac{1}{\lambda^2}}{u}\right)} \tag{13}$$

$$ln\, M^{Z} \geq ln\left\{\frac{ln\left[\frac{1}{2(M-1)}\right]}{ln\left(\frac{u-\frac{1}{\lambda^2}}{u}\right)}\right\}$$

$$Z \geq \frac{1}{ln\, M} \cdot ln\left\{\frac{ln\left[\frac{1}{2(M-1)}\right]}{ln\left(\frac{u-\frac{1}{\lambda^2}}{u}\right)}\right\}. \tag{14}$$

where

$$u \leq C_2^M \cdot (n+p) \leq C_2^M \cdot (2n) \leq M \cdot (M-1) \cdot n \tag{15}$$

and $n = \max(n,p)$. Since we wish to derive the absolute upper bound of u, it will be reasonable to use $n = \max(n,p)$ instead of $n = \min(n,p)$ in the above result. We deem the above equation to be the absolute upper bound of u. The above Eqn.(14) is the lower bound solution of Z, and according to Eqn.(13), we also can derive the capacity (M) as follows,

$$M \geq \left\{\frac{ln\left[\frac{1}{2(M-1)}\right]}{ln\left(\frac{u-\frac{1}{\lambda^2}}{u}\right)}\right\}^{Z-1} \tag{16}$$

where

$$u = M \cdot (M\text{-}1) \cdot n.$$

3. SIMULATION ANALYSIS

In order to verify the capacity analysis described in Section 2.3, we use computer programs to produce the values among M, Z, and n for λ equal to 2, 5, 10, and 100 for the lower bound solution of Z. The above analyses are plotted in Fig. 1, Fig. 2, Fig. 3 and Fig. 4, respectively (The legends represent the values of Z). After the lower bound solution of Z is derived, the capacity of the fuzzy data recall using the PBHC can be computed by Eqn.(16). Fig. 5, Fig. 6, and Fig. 7 are the relationships of capacity (M) vs. n in the lower bound solution of M. In Fig. 5, $\lambda = 10$, $Z = 3$; In Fig. 6, $\lambda = 100$, $Z = 4$; and in Fig. 7, $\lambda = 300$, $Z = 4$. From the figures, we can see that the fuzzy data recall using the PBHC provides a significantly high capacity of storage for patterns.

Example 1. We can use the result of this research for pattern recognition. We tend to use the PBHC to store and recall a set of 7 × 11 fuzzy data composed of twenty-six different pattern pairs (English letters, upper case and lower case). We next apply the evolution equation in Section 2.1 and the lower bound of solution of Z in Section 2.3. In Fig. 8 and Fig. 9, we present every pattern pair with $n = p = 77$ to this network, and we find that just one iteration is required for every capital letter to recall its lower case letter correctly, and vice versa.

4. CONCLUSION

The fuzzy data recall using the PBHC provides a significantly high capacity of storage for patterns. It utilizes a fuzzy scheme to magnify the capacity. The proposed energy function ensures that every stored pattern pair is located in a local minimum of the energy surface. The capacity of the PBHC is estimated, so the size of the PBHC can be predetermined by the demand of capacity.

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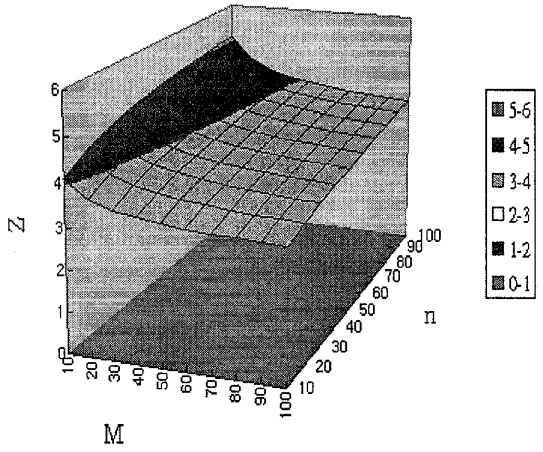

Fig. 1 The relationship of M, Z and n for λ=2

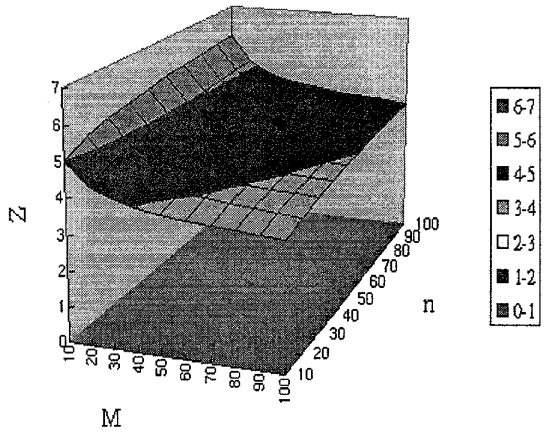

Fig. 2 The relationship of M, Z and n for λ=5

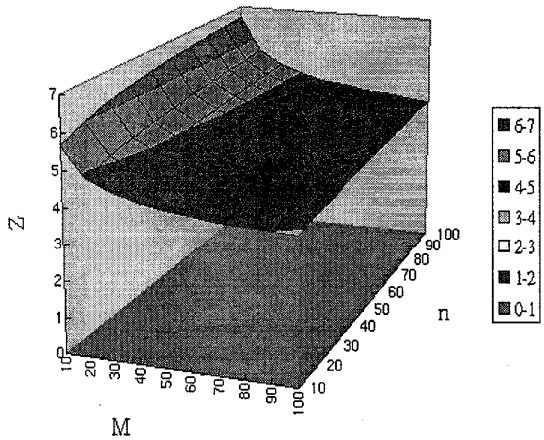

Fig. 3 The relationship of M, Z and n for λ=10

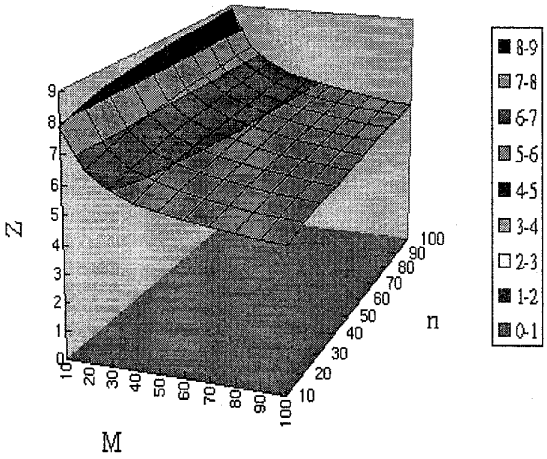

Fig. 4 The relationship of M, Z and n for λ=100

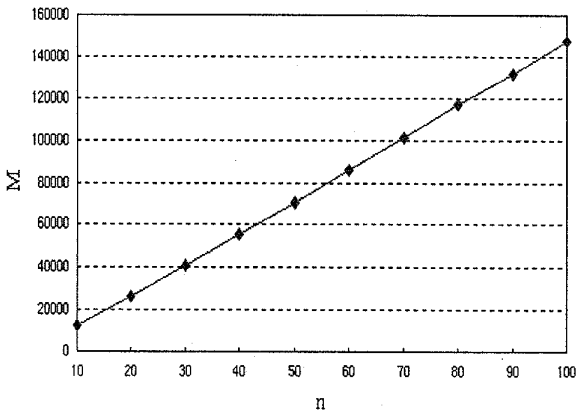

Fig. 5 The capacity of PBHC in the worst case (λ=10, Z=3)

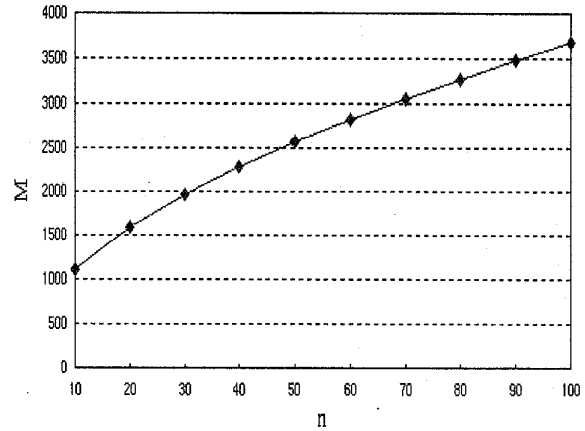

Fig. 6 The capacity of PBHC in the worst case (λ=100, Z=4)

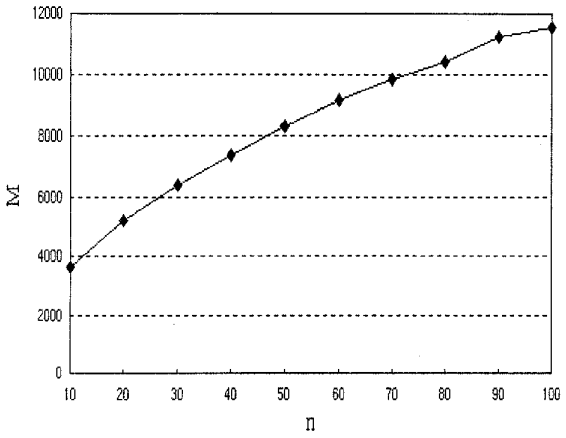

Fig. 7 The capacity of PBHC in the worst case (λ=300, Z=4)

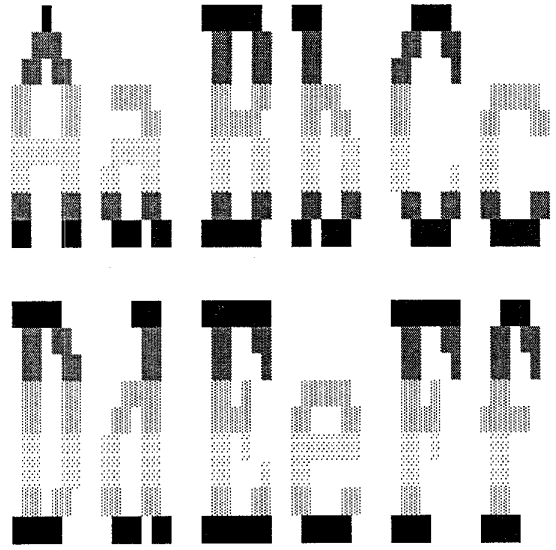

Fig. 8 Pattern recognition examples (M=26, n=77, p=77)

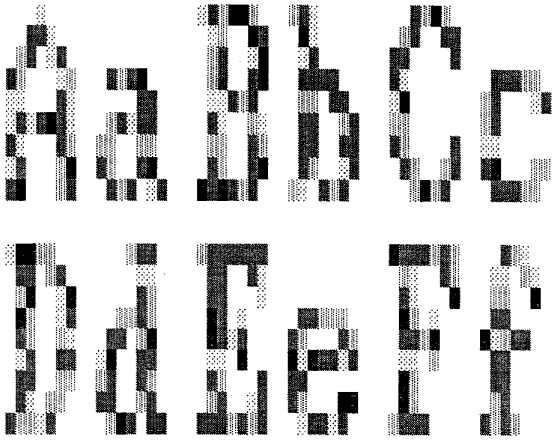

Fig. 9 Pattern recognition examples (M=26, n=77, p=77)