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The Decision Making Rule of Multiple Exponential Bidirectional Associative Memories [1]

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Abstract

A method for modeling the learning of belief combination in evidential reasoning using a neural network is presented . A centralized network composed of multiple exponential bidirectional associative memories (eBAMs) sharing a single output array of neurons is proposed to process the uncertainty management of many pieces of evidence simultaneously. The stability of the proposed multiple eBAM network is proved. The sufficient condition to recall a stored pattern pair is discussed. Most important of all, a majority rule of decision making in presentation of multiple evidence is also found by the study of signal-noise-ratio (SNR) of multiple eBAM network. The result is coherent with the intuition of reasoning.

1 Introduction

Neural networks have been drawing increasing interest as powerful tools to solve different tasks of artificial intelligence. An *associative memory* is one type of neural network which essentially is a single functional layer or slab that associates one set of vectors with another set of vectors. Kosko [4] proposed a two-level nonlinear network, *bidirectional associative memory* (BAM), which extends a one-directional process to a two-directional process. Jeng [3] and Wang [8], respectively, then generalized the concept of storing information in the exponential BAM.

Among the problems of evidential reasoning, conflicts caused by sequential programming and partial dependency are pretty hard to be fully resolved [6]. The basic reason is all of the traditional methods for evidential reasoning are developed for two pieces of evidence. Thus, when there are more than two pieces of evidence, conflicts will happen if the combination orders are different [6]. Wang *et al.* pointed out the importance of simultaneously processing many pieces of evidence [6], and they further proposed a method using multiple BAM structure to handle the demand of combining many evidence at the same time [7]. However, due to the inherently poor capacity of BAM [3], [8], obviously the multiple BAM network would be limited to a foreseeable degree of processing capability. We propose a multiple eBAM network to increase the processing capability of reasoning many evidence. We also discuss the majority rule of decision making for handling many evidence at the same time. The majority rule means if more than half of the presented evidence support one hypothesis, though the rest of the presented evidence don't, the belief combination of all of these evidence must be dominated by the hypothesis. This rule is intuitively in accordance with the human reasoning.

2 Framework of Multiple Exponential BAMs Network

2.1 Evolution equations

As shown in Fig. 1, the multi-eBAM network is constructed with L single eBAMs which share a common output array of neurons. In each clock, the input vectors are presented at the input array of neurons, respectively. Suppose we are given N training sample pairs to the qth eBAM of the network, which are $\{(A_{q1}, B_1), (A_{q2}, B_2), ..., (A_{qN}, B_N)\}$, where $A_{qi} = (a_{qi1}, a_{qi2}, ..., a_{qin}), B_i = (b_{i1}, b_{i2}, ..., b_{ip})$. Let X_i and Y_i be the bipolar mode of the training pattern pairs, A_{qi} and B_i, respectively. That is, $X_{qi} \in \{-1,1\}^n$ and $Y_i \in \{-1,1\}^p$. Thus, we use the following evolution equations in the recall process of the multi-eBAM network :

$$y_k = sgn\left(\sum_{q=1}^{L}\sum_{i=1}^{N} y_{ik} b^{X_{qi}\cdot X_q}\right), \quad x_{qk} = sgn\left(\sum_{i=1}^{N} x_{qik} b^{Y_i \cdot Y}\right) \tag{1}$$

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where b is a positive number, $b > 1$, "$\cdot$" represents the inner product operator, x_k and x_{qik} are the kth bits of X and the X_{qi}, respectively, and y_k and y_{ik} are for Y and the Y_i, respectively.

2.2 Energy function and stability

Since every stored pattern pair should produce a local minimum on the energy surface [8] in order to be retrieved correctly, the multi-eBAM network's overall energy function is defined as

$$E = \sum_q E(X_q, Y) = -\sum_{q=1}^{L}\sum_{i=1}^{N}(b^{X_{qi}\cdot X_q} + b^{Y_i \cdot Y}) \tag{2}$$

Assume $E(X_q', Y)$ is the energy of next state in which Y stays the same as in the previous state, and all of the other eBAMs stay at the same state as before. Hence, $\triangle E_{xq} = -\sum_{i=1}^{N} b^{X_{qi}\cdot X_q'} - (-\sum_{i=1}^{N} b^{X_{qi}\cdot X_q})$. Assume the ith pair is the target of the recall process for the qth eBAM. Let d_{qxi} be the Hamming distance between X_q and X_{qi}, d_{qxi}' the Hamming distance between the X_q' and X_{qi}. Hence the $\triangle E_{xq}$ can be modified to be :

$$\triangle E_{xq}' = -\sum_{i=1}^{N}\log_b(b^{n-2d_{qxi}'}) + \sum_{i=1}^{N}\log_b(b^{n-2d_{qxi}}) = -\sum_{i=1}^{N}\sum_{k=1}^{n}(x_{qk}' - x_{qk})x_{qik}, \tag{3}$$

Note that log is used, which is a monotonic function. From the recall process shown by Eqn.(1) and Eqn.(3), the $\triangle E_{xq} < 0$ is ensured. This result was also proved by Jeng *et al.* [3]. Because Eqn.(1) makes $(x_{qk}' - x_{qk})x_{qik}$ always nonnegative such that $\triangle E_{xq}' \leq 0$, and then $E_{xq} \leq 0$ can be directly derived [8]. Obviously, it also holds for the other case : $E(X_q, Y') \leq E(X_q, Y)$ if the pair is heading for a stored pair, (X_{qi}, Y_i). Since the $E(X_q, Y)$ is bounded by $-N(b^n + b^p) \leq E(X, Y) \leq -N(b^{-n} + b^{-p})$ for all X_q's and Y, the energy of the multi-eBAM network will converge to a stable local minimum.

2.3 A Majority Rule for the Multi-eBAM Network

A. The majority rule of a special case

Every single eBAM tends to store their own pattern pairs in the local minimums of their network, respectively. However, not every single eBAM will agree to have a common output pattern. Hence, we formulate the entire problem as follows : Given a multi-eBAM network composed of L single eBAMs, what is the minimal majority factor k, $k \in [0, 1]$, to make kL eBAMs, which are vowing a common output pattern and the other eBAMs are not, dominate the common output? Note that in fact the kL denotes an integer, $Ceiling(kL)$, which is the smallest integer larger than kL.

Before we discuss the lower bound of kL, we have to study an extreme case in which a upper bound of kL will be derived. Assume the pattern pairs, $(X_{11}, Y_r), (X_{21}, Y_r), \ldots, (X_{kL1}, Y_r)$, are encoded in 1st to kLth eBAMs, respectively, and pattern pairs, $(X_{(kL+1)1}, Y_s), \ldots, (X_{L1}, Y_s)$, are stored in $(kL+1)$th to Lth eBAMs, respectively. Thus, when input patterns, $X_{11}, X_{21}, \ldots, X_{L1}$, are presented at the input array of each individual eBAM, the Y_r is the output pattern that we are looking for, i.e., it is deemed as the signal. By the SNR approach [1], [3], [8] and the evolution equations (1), we are aware of the following facts :

$$\sum_{q=1}^{L}\sum_{i=1}^{N} y_{ik} b^{X_{qi}\cdot X_q} = \sum_{q=1}^{kL}(y_{rk}b^n + \sum_{i\neq r} y_{ik} b^{X_{qi}\cdot X_{qr}}) + \sum_{q=kL+1}^{L}(y_{sk}b^n + \sum_{i\neq s} y_{ik} b^{X_{qi}\cdot X_{qs}}) \tag{4}$$

By our previous assumptions, only the first term in the above equation is the signal we wish to observe at the output array of processing units. As for the rest terms, they are the undesired noise. Therefore, we can derive the signal power is $S = \sum_{q=1}^{kL} b^{2n} = kLb^{2n}$, and the largest power of noise, which means all of the rest $(1-k)L$ eBAMs support another output pattern Y_s, is $N = (1-k)Lb^{2n} + L(N-1)b^{2(n-2)}$. In the noise power equation, we assume not only all of the rest $(1-k)L$ eBAMs support another output pattern, but also this pattern is the closest pattern to the desired one. If the desired output is intended to be recalled, then the sufficient condition is the $S > N$ according to the SNR approach. Thus we can conclude the lower bonds for this definite recall condition of k is

$$kLb^{2n} > (1-k)Lb^{2n} + L(N-1)b^{2(n-2)} \Rightarrow k > \frac{1}{2} + \frac{N-1}{2b^4} \quad (5)$$

Note that this lower bound of k means any k bigger than this threshold can force the output pattern to be the common desired output pattern of the kL eBAMs in the network. However, if the bound of Eqn.(5) is larger than 1, it means even all of the eBAMs support one output pattern, there is no guarantee to recall this common pattern.

B. The majority rule of the general case

In the above extreme case, we assume all of the rest $(1-k)L$ eBAMs support another same output pattern which is only one bit Hamming distance away from the desired pattern. Generally speaking, however, most of the reasoning problems won't be this special. We will consider a general case in which kL eBAMs still support a common output pattern, but the rest $(1-k)L$ eBAMs don't support the same output pattern, i.e., they individually support their own output patterns, respectively. Basing upon this assumption, then we can derive the following results.

$$\sum_{q=1}^{L}\sum_{i=1}^{N} y_{ik} b^{X_{qi}\cdot X_q} = kL\cdot y_{rk}b^n + (1-k)L\cdot y_{sk}b^n + \sum_{q=1}^{kL}\sum_{i\neq r} y_{ik}b^{X_{qi}\cdot X_{qr}} + \sum_{q=kL+1}^{L}\sum_{i\neq s} y_{ik}b^{X_{qi}\cdot X_{qs}} \quad (6)$$

The 3rd and 4th terms of the above equation can be analyzed by the SNR approach proposed by Wang [8]. Hence, the power of the 3rd and 4th terms are, respectively,

$$N_3 = 2(\frac{1}{2})^{n-1}\frac{b^{2n}(1+b^{-4})^{n-1}}{b^4}\cdot kL(N-1), \qquad N_4 = 2(\frac{1}{2})^{n-1}\frac{b^{2n}(1+b^{-4})^{n-1}}{b^4}\cdot(1-k)L(N-1) \quad (7)$$

The SNR of this case can be further derived,

$$SNR = \frac{k}{(1-k)+(N-1)\cdot 2(\frac{1}{2})^{n-1}\frac{(1+b^{-4})^{n-1}}{b^4}} \quad (8)$$

If the common desired output pattern must be recalled, then the sufficient condition is the SNR must be greater than 1. Therefore, we conclude the above discussion of a majority rule of multi-eBAM network with the following theorem:

Theorem of the majority rule for a multi-eBAM network : Given a multi-eBAM network with L single eBAMs, kL eBAMs support a same common output pattern, where $k \in [0,1]$. The condition for the output pattern of the network is the same as the one supported by the kL eBAMs is

$$k > \frac{1}{2} + \frac{1}{2\cdot SNR_{eBAM}} \quad (9)$$

where $SNR_{eBAM} = \frac{2^{n-1}b^4}{2(N-1)(1+b^{-4})^{n-1}}$ according to Wang's analysis [8]. If the lower bound k in Eqn.(9) is larger than 1, then it means even all of the single eBAMs in the network support one output pattern, there is no guarantee to recall this output pattern.

By the above theorem, please note because the SNR_{eBAM} is usually very large, the lower bound of k can be simplified to be $\frac{1}{2}$ which complies the human intuition. That is, if more than 50% of the evidence supports a hypothesis, then the reasoning result most likely would be the same as this hypothesis.

3 Simulation Analysis

Example 1. We construct a series of multi-eBAM networks, $L = 3$ to $L = 31$, $n = p = 8$, $b = e$ to verify the k prediction of the majority rule. In every multi-eBAM simulation, the number of stored pattern pairs for each single eBAM is also varied from $N = 3$ to $N = 99$. In this simulation, all of the pattern pairs are randomly generated. The result is the majority rule holds in every case, even in the worse case, $L = 31, N = 99$, in which the predicted $k = 0.515923$, and $16/31 = 0.516129 > 0.515923$. The relation between k and N is illustrated in Fig. 2, while the minimal k selected in networks with different number of eBAMs is shown in Fig. 3. For the sake of comparison, we also repeat the simulation for the special case. The minimal k in the special case is shown in Fig. 4.

4 Conclusion

A multi-eBAM neural network has been introduced for the belief combination in evidential reasoning. It is proved to be bidirectionally stable, which ensures the model's ability to reach a local energy minimum. The sufficient conditions for a multi-eBAM network guarantee the network to recover a specific, predetermined pattern pair from a list of choices. Most important of is the theorem of the majority rule of the network proves this neural network complys with the intuition of human reasoning. Two majority rules and their respective bounds for the majority factor, k, are presented. These rules will help researchers to use and predict the result of evidential reasoning. This network provides the ability to process many evidence at the same time reaching a consented hypothesis.

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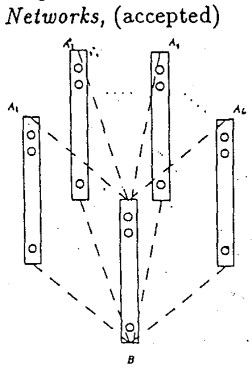

Fig. 1 The configuration of a multi-eBAM neural network.

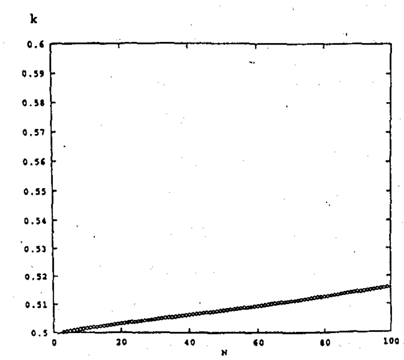

Fig. 2 The lower bound of k with different N.

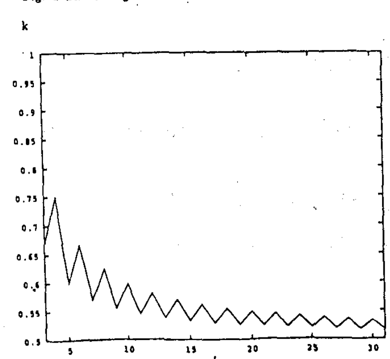

Fig. 3 The minimal k in different networks.

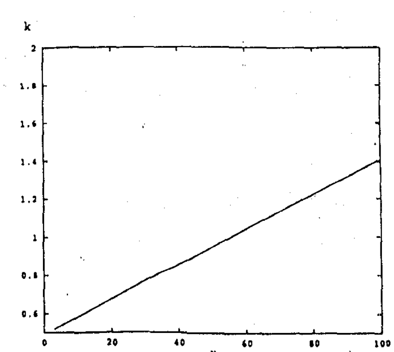

Fig. 4 The lower bound of k in the special case.