An Interference Cancellation Scheme for Carrier Frequency Offsets Compensation in the Uplink of OFDMA Systems

Sen-Hung Wang[1], Chih-Peng Li[2], Chua-Chin Wang[3]

[1,3]Department of Electrical Engineering

[2]Institute of Communications Engineering

National Sun Yat-Sen University

[1]m933070005@student.nsysu.edu.tw, [2]cpli@mail.nsysu.edu.tw, [3]ccwang@ee.nsysu.edu.tw.

Abstract

A successive interference cancellation (SIC) structure is proposed for multi-user interference cancellation (MUI) due to carrier frequency offsets (CFOs) in the uplink of orthogonal frequency division multiple access (OFDMA) systems. The proposed architecture adopts a circular convolution to suppress the impacts caused by CFOs. This paper demonstrates that, with 2 iterations, the SIC has better performance than that of the parallel interference cancellation (PIC) structure. However, system complexity is only 1/2K, where K is the number of users in the system. This study also shows that system complexity can be significantly reduced if proper approximation is made.

Keywords - Orthogonal frequency division multiple access (OFDMA), carrier frequency offset, multi-user interference (MUI), circular convolution, successive interference cancellation (SIC).

I. INTRODUCTION

Recently, orthogonal frequency division multiple access (OFDMA) scheme has been adopted in IEEE 802.16-2004 for wireless metropolitan area network (WMAN) applications [1-2], and has been proposed in cable television (CATV) data networks [3] and satellite communications [4]. In OFDMA, closely spaced and overlapped subcarriers are divided into groups and assigned to multiple users for simultaneous transmissions.

OFDMA inherits the defects from Orthogonal Frequency Division Multiplexing (OFDM) and is very sensitive to inaccurate frequency reference. In OFDMA systems, sampling clock frequency mismatch, carrier frequency offsets (CFOs) due to poor oscillator alignments and Doppler effects, and timing delay caused by multi-path and non-ideal synchronization will seriously destroy the orthogonality among subcarriers. Moreover, they introduce the inter-carrier interference (ICI) and the multi-user interference (MUI) [5-8].

Sampling clock frequency incongruity can be compensated by channel estimation. For time delay discrepancy, guard intervals, a global positioning system (GPS), and the downlink information can be used to mitigate their influences. However, the carrier frequency is on the order of gigahertz, and the CFOs are normally on the order of kilohertz. Mitigating the impact caused by CFOs is the most critical challenge among the above mentioned problems.

A variety of schemes have been proposed to alleviate the impact of ICI in OFDM systems [9-12]. In particular, the impact of ICI is substantially lessened by the ICI self-cancellation scheme [9], which modulates one data symbol onto a group of subcarriers with predefined weighting coefficients. The major drawback of the ICI self-cancellation scheme is that the spectrum efficiency dramatically decreases as the number of subcarriers in a group increases. A number of ICI cancellation schemes and their analyses can be found in [11].

A carrier frequency offset compensation scheme is proposed by *Choi et al.* [5], in which CFOs are corrected after the discrete Fourier transform (DFT) using circular convolution. This circular convolution scheme is adopted by *Huang and Letaief* [6] to compensate CFOs in the uplink of OFDMA systems and then the multi-user interference is reconstructed using a parallel interference cancellation (PIC) architecture. The structure proposed in [6] substantially diminishes the influence of MUI. However, the proposed scheme has to process all the users in parallel, leading to an extremely high cost in hardware implementation.

To reduce the system complexity of the PIC architecture in mitigating MUI, a successive interference cancellation (SIC) structure is proposed in this paper for the uplink of OFDMA systems. In the proposed scheme, as adopted by *Huang and Letaief* [6], each user's transmitted signal is restored using the circular convolution scheme proposed by C*hoi et al.* [5]. After the detection process, the interference terms are reconstructed using three steps. First, the information bits are modulated. Then the channel effects are reconstructed. Finally, the effects caused by CFOs are rebuilt using the circular convolution scheme. Subsequently, the interference terms are subtracted from the received signal. The interference cancellation processes are made one user after another. Simulation results demonstrate, with only one or two iterations, the performance of the SIC structure is comparable to or better than that of the PIC. However, the complexity of the SIC structure is only 1/2*K*, where *K* is the number of users in system.

This paper is organized as follows. Section II describes the system model. Section III delineates the proposed SIC architecture and the operation principles. The system signal-to-interference ratio (SIR) and complexity are analyzed in section IV and simulation results are shown in section V. Section VI concludes this investigation.

II. SYSTEM MODEL

The system under investigation is the uplink of an OFDMA system with *N* subcarriers and *K* users, where each user terminal (UT) communicates with the base station (BS) through an independent multi-path channel, as shown in Fig. 1. The *N* subcarriers are equally divided into *K* groups and assigned to *K* users. Since one subcarrier is only allocated to one user, each user has N/K subcarriers. After inverse discrete Fourier transform (IDFT) and guard-interval addition, the time-domain discrete signal of the *k*th user is given by

$$x_n^k = \frac{1}{\sqrt{N}} \sum_{i \in \Gamma_k} X_i^k e^{\frac{j2\pi ni}{N}}, \quad -N_g \le n \le N-1 \tag{1}$$

where X_i^k is the *k*th user's information symbol at the *i*th subcarrier, $i \in \Gamma_k$, Γ_k is the set of subcarriers assigned to the *k*th user, and N_g is the length of the guard-interval.

After passing through each individual user's channel, the *k*th user's received signal can be represented as

$$y_n^k = x_n^k \otimes h_n^k \tag{2}$$

where "$\otimes$" denotes linear convolution, and h_n^k is the *k*th user's channel impulse response (CIR).

At the BS, the total received baseband signal, which is corrupted by the additive white Gaussian noise (AWGN), from all the users is given by

$$r_n = \sum_{k=1}^{K} y_n^k \cdot e^{\frac{j2\pi\varepsilon_k n}{N}} + z_n, \ -N_g \le n \le N-1 \tag{3}$$

where ε_k is the kth user's CFO normalized by the subcarrier spacing, and z_n is the AWGN.

After the guard-interval removal and DFT processing of the total received signal given in (3), the received signal on the ith subcarrier is expressed as

$$R_i = \mathrm{DFT}_N\left(\sum_{k=1}^{K} y_n^k \cdot e^{\frac{j2\pi\varepsilon_k n}{N}} + z_n\right), \qquad 0 \le n \le N-1 \tag{2}$$

$$= \sum_{k=1}^{K}\sum_{m_k \in \Gamma_k}\left(X_{m_k}^k \cdot H_{m_k}^k\right)\cdot C_{(m_k-i)\bmod N}^{k}{}' + Z_i, \tag{4}$$

where $\mathrm{DFT}_N(\cdot)$ denotes the N-point DFT. X_i^k, H_i^k, and Z_i are x_n^k, h_n^k, and z_n after the N-point DFT operation, respectively. In addition,

$$\mathrm{DFT}_N\left(f(n)\right) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} f(n)\cdot\exp\left(-\frac{j\cdot 2\pi ni}{N}\right) \tag{5}$$

for any function $f(n)$, and

$$C_q^k{}' = \frac{\sin\left(\pi\left(q+\varepsilon_k\right)\right)}{N\cdot\sin\left(\frac{\pi}{N}\left(q+\varepsilon_k\right)\right)}\cdot\exp\left(j\pi\left(1-\frac{1}{N}\right)\left(q+\varepsilon_k\right)\right). \tag{6}$$

For convenience, (4) can be written into a vector form as follows:

$$\begin{aligned}\mathbf{R} &= \sum_{k=1}^{K}\left(\mathbf{H}^k\cdot\mathbf{X}^k\right)\otimes_N \mathbf{C}^{k}{}' + \mathbf{Z}\\ &= \left(\mathbf{H}^p\cdot\mathbf{X}^p\right)\otimes_N \mathbf{C}^{p}{}'\\ &\quad+\sum_{\substack{k=1\\k\neq p}}^{K}\left(\mathbf{H}^k\cdot\mathbf{X}^k\right)\otimes_N \mathbf{C}^{k}{}' + \mathbf{Z}\\ &= \mathbf{D} + \mathbf{I}_{MUI} + \mathbf{Z},\end{aligned} \tag{7}$$

where

$$\mathbf{R} = \left[R_0, R_1, \cdots, R_{N-1}\right]^T \tag{8}$$

$$\mathbf{X}^k = \left[X_0^k, X_1^k, \cdots, X_{N-1}^k\right]^T \tag{9}$$

$$\mathbf{H}^k = \mathrm{diag}\left(H_0^k, H_1^k, \cdots, H_{N-1}^k\right) \tag{10}$$

$$\mathbf{Z} = \left[Z_0, Z_1, \cdots, Z_{N-1}\right]^T \tag{11}$$

$$\mathbf{C}^{k}{'} = \left[C_0^k{'}, C_1^k{'}, \cdots, C_{N-1}^k{'}\right] \tag{12}$$

, $\mathrm{diag}(\cdot)$ denotes the diagonal matrix, " $\otimes_N$ " denotes N-point circular convolution, and $[\cdot]^T$ denotes the transpose operation. The first term $\mathbf{D}$ in (7) is the desired user's signal plus inter-carrier interference, the second term $\mathbf{I}_{MUI}$ is the multi-user interference caused by CFOs, and the third term $\mathbf{Z}$ is the additive white Gaussian noise.

According to (7), the effects caused by the CFOs can be expressed as a circular convolution with the transmitted signal. Both the inter-carrier interference and the multi-user interference can be efficiently mitigated by properly applying the circular convolution technique to the received signals.

III. THE PROPOSED SUCCESSIVE INTERFERENCE CANCELLATION ARCHITECTURE

The successive interference cancellation architecture proposed in this investigation for the uplink of OFDMA systems is depicted in Fig. 2. In principle, the MUI cancellation process of the proposed SIC structure consists of several iterations and the signal has to be sequentially processed for all the users in each iteration. The signal processing of each user comprises two stages: the detection stage and the interference regeneration stage. In the detection stage, the circular convolution is first applied on a given user's signal to compensate the effect caused by CFOs and then the user's information bits are detected. In the interference regeneration stage, the circular convolution is adopted again to reconstruct the effect caused by CFOs so that the MUI of a given user can be regenerated. Then, the MUI of the given user is subtracted from the received signal. The MUI cancellation is sequentially made for all the users in each iteration. Detail operations of the SIC architecture are delineated in the following.

To obtain the best performance of the proposed architecture, the users have to be sorted by their CFOs from the biggest to the smallest. Because the biggest one causes most of MUI and the detection scheme aims at the ICI. It can obtain the best signal-to-interference ratio (SIR). After sorting, the users' indices and the corresponding CFOs are expressed as $\mathbf{s} = [s_1, s_2, \cdots, s_k]$ and $\boldsymbol{\varepsilon}' = [\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_k]$, respectively. Detection of the information bits and regeneration of the MUI are made one user after another according to the sequence $\mathbf{s}$.

In Fig. 2, $\mathbf{R}_{i,p}$ denotes the received signal of the *p*th user (with user index s_p) in the *i*th iteration ($i = 1, 2, \cdots$). In particular, $\mathbf{R} = \mathbf{R}_{1,1}$. In addition, the regenerated MUI of the *p*th user in the *i*th iteration is given by $\hat{\mathbf{I}}_{i,p}$.

In the detection stage of the *p*th user (with user index s_p) in the *i*th iteration, the received signal $\mathbf{R}_{i,p}$ is multiplied by a mask $\mathbf{A}^p$, where $\mathbf{A}^p$ is a $N \times N$ diagonal matrix used to extract the *p*th user's subcarriers. The *n*th diagonal element of $\mathbf{A}^p$ is equal to one if $n \in \Gamma_p$, where Γ_p is the set of subcarriers assigned to the *p*th user. All the other elements are equal to zero. A *N*-point circular convolution with $\mathbf{C}^p$ is applied to the output of the mask operation to compensate the *p*th user's CFO ε_p as given by

$$\tilde{\mathbf{Y}}^p = \left(\mathbf{A}^p \cdot \mathbf{R}_{i,p}\right) \otimes_N \mathbf{C}^p \tag{13}$$

where

$$\mathbf{C}^p = \left[C_0^p, C_1^p, \cdots, C_{N-1}^p\right] \tag{14}$$

and

$$C_q^p = \frac{\sin\left(\pi\left(q - \varepsilon_p\right)\right)}{N \cdot \sin\left(\frac{\pi}{N}\left(q - \varepsilon_p\right)\right)} \cdot \exp\left(j\pi\left(1 - \frac{1}{N}\right)\left(q - \varepsilon_p\right)\right). \tag{15}$$

After CFO compensation, the *p*th user's channel effects are corrected by

$$\tilde{\mathbf{X}}^{p} = \left(\tilde{\mathbf{H}}^{p}\right)^{-1} \cdot \tilde{\mathbf{Y}}^{p} \tag{16}$$

where $\tilde{\mathbf{H}}^{p}$ is the pth user's channel response and $(\cdot)^{-1}$ denotes the inverse operation. Finally, the resulted signal $\tilde{\mathbf{X}}^{p}$ is sent for demodulation to obtain the pth user's estimated information bits $\hat{\mathbf{a}}^{p}$, which completes the detection stage of the pth user in the ith iteration.

In the interference regeneration stage, the information bits $\hat{\mathbf{a}}^{p}$ are adopted to regenerate the pth user's interference. First, the same modulation scheme as that used at the transmitter is adopted to modulate the pth user's information bits and the result is denoted as $\tilde{\mathbf{X}}^{p}{'}$. Second, the pth user's channel effects are reconstructed as given by:

$$\tilde{\mathbf{Y}}^{p}{'} = \tilde{\mathbf{H}}^{p} \cdot \tilde{\mathbf{X}}^{p}{'} \tag{17}$$

Then, the effect caused by the pth user's CFO is reconstructed by applying the N-point circular convolution with $\mathbf{C}^{p}{'}$ as given in (12), which is followed by the mask operation $\mathbf{A}^{p}{'}$ to generate the multi-user interference caused by the pth user:

$$\hat{\mathbf{I}}_{i,p} = \mathbf{A}^{p}{'} \cdot \left(\tilde{\mathbf{Y}}^{p}{'} \otimes_{N} \mathbf{C}^{p}{'}\right) \tag{18}$$

where $\mathbf{A}^{p}{'}$ is a $N \times N$ diagonal matrix, which is used to repel the pth user's subcarriers. The nth diagonal element of $\mathbf{A}^{p}{'}$ is equal to zero if $n \in \Gamma_{p}$ and all the other elements are equal to one. In other words, $\mathbf{A}^{p} + \mathbf{A}^{p}{'} = \mathbf{I}$, where $\mathbf{I}$ is an identity matrix. Finally, the received signal of the (p+1)th user in the ith iteration is obtained by

$$\mathbf{R}_{i,p+1} = \mathbf{R}_{i,p} - \hat{\mathbf{I}}_{i,p} + \hat{\mathbf{I}}_{i-1,p} \tag{19}$$

It is worthy of noting that $\hat{\mathbf{I}}_{0,p} = 0$. This MUI cancellation process has to be sequentially made for all the K users in system. For the Kth user, since all the K users' MUIs have been eliminated, the index p has to be reset to one and the iteration index i has to be increased by one to begin a new iteration. Of course, the system performance increases with the number of iterations, which in turn increase the processing time. Simulation results demonstrate the performance of the SIC surpasses that of the PIC in two iterations.

IV. ANALYSES OF SIGNAL-TO-INTERFERENCE RATIO AND COMPLEXITY

In this section, an analysis of signal-to-interference ratio (SIR) and the complexity of the proposed scheme are provided. Throughout this section, we assume that the multi-user interference and the inter-carrier interference are independent and Gaussian distributed. This is a reasonable assumption as long as the number of sub-carriers for each user is large enough.

A. ANALYSIS OF SIGNAL-TO-INTERFERENCE RATIO

Considering an additive white Gaussian noise (AWGN) channel, the desired signal, after the circular convolution, of the rth sub-carrier for the first cancellation in (7) is given by:

$$\begin{aligned}
\mathrm{D}(r,1) &= Z_r^1 \sum_{i\in\Gamma_1} I\left(r-i+\varepsilon_1\right)\cdot I\left(-r+i-\varepsilon_1\right) \\
&= Z_r^1 \sum_{i\in\Gamma_1} \frac{1}{N^2} \frac{\sin\left(\pi\left(r-i+\varepsilon_1\right)\right)}{\sin\left(\frac{\pi}{N}\left(r-i+\varepsilon_1\right)\right)} \exp\left(j\left(\frac{N-1}{N}\right)\left(r-i+\varepsilon_1\right)\right) \\
&\quad \cdot \frac{\sin\left(-\pi\left(r-i+\varepsilon_1\right)\right)}{\sin\left(-\frac{\pi}{N}\left(r-i+\varepsilon_1\right)\right)} \exp\left(-j\left(\frac{N-1}{N}\right)\left(r-i+\varepsilon_1\right)\right) \\
&= \frac{1}{N^2} Z_r^1 \sum_{i\in\Gamma_1} \frac{\sin^2\left(\pi\left(r-i+\varepsilon_1\right)\right)}{\sin^2\left(\frac{\pi}{N}\left(r-i+\varepsilon_1\right)\right)},
\end{aligned} \tag{20}$$

where Z_r^1 is the first user's modulated symbol of the rth sub-carrier, $r\in\Gamma_1$, and

$$I(v) = \frac{1}{N}\cdot\frac{\sin(\pi v)}{\sin\left(\frac{\pi v}{N}\right)} \times \exp\left(j\left(\frac{N-1}{N}\right)v\right). \tag{21}$$

The inter-carrier interference term of the rth sub-carrier for the first cancellation is given by:

$$
\begin{aligned}
\mathrm{I}_{\mathrm{ICI}}(r,1) &= \sum_{i\in\Gamma_1}\sum_{\substack{m_1\in\Gamma_1\\ m_1\neq r}} Z_{m_1}^{1}\cdot I\left(m_1-i+\varepsilon_1\right)\cdot I\left(-r+i-\varepsilon_1\right)\\
&= \frac{1}{N^2}\sum_{i\in\Gamma_1}\sum_{\substack{m_1\in\Gamma_1\\ m_1\neq r}} Z_{m_1}^{1}\cdot\frac{\sin\left(\pi\left(m_1-i+\varepsilon_1\right)\right)}{\sin\left(\frac{\pi}{N}\left(m_1-i+\varepsilon_1\right)\right)}\cdot\exp\left(j\left(\frac{N-1}{N}\right)\left(m_1-i+\varepsilon_1\right)\right)\\
&\quad\cdot\frac{\sin\left(\pi\left(-r+i-\varepsilon_1\right)\right)}{\sin\left(\frac{\pi}{N}\left(-r+i-\varepsilon_1\right)\right)}\cdot\exp\left(j\left(\frac{N-1}{N}\right)\left(-r+i-\varepsilon_1\right)\right)\\
&= \frac{1}{N^2}\sum_{i\in\Gamma_1}\sum_{\substack{m_1\in\Gamma_1\\ m_1\neq r}} Z_{m_1}^{1}\cdot\frac{\sin\left(\pi\left(m_1-i+\varepsilon_1\right)\right)\sin\left(\pi\left(-r+i-\varepsilon_1\right)\right)}{\sin\left(\frac{\pi}{N}\left(m_1-i+\varepsilon_1\right)\right)\sin\left(\frac{\pi}{N}\left(-r+i-\varepsilon_1\right)\right)}\\
&\quad\cdot\exp\left(j\left(\frac{N-1}{N}\right)\left(m_1-r\right)\right).
\end{aligned}
\tag{22}
$$

The multi-user interference term of the rth sub-carrier for the first cancellation is given by:

$$
\begin{aligned}
\mathrm{I}_{\mathrm{MUI}}(r,1) &= \sum_{i\in\Gamma_1}\sum_{k=2}^{K}\sum_{m_k\in\Gamma_k} Z_{m_k}^{k}\cdot I\left(m_k-i+\varepsilon_k\right)\cdot I\left(-r+i-\varepsilon_1\right)\\
&= \frac{1}{N^2}\sum_{i\in\Gamma_1}\sum_{k=2}^{K}\sum_{m_k\in\Gamma_k} Z_{m_k}^{k}\\
&\quad\cdot\frac{\sin\left(\pi\left(m_k-i+\varepsilon_k\right)\right)}{\sin\left(\frac{\pi}{N}\left(m_k-i+\varepsilon_k\right)\right)}\exp\left(j\left(\frac{N-1}{N}\right)\left(m_k-i+\varepsilon_k\right)\right)\\
&\quad\cdot\frac{\sin\left(\pi\left(-r+i-\varepsilon_1\right)\right)}{\sin\left(\frac{\pi}{N}\left(-r+i-\varepsilon_1\right)\right)}\exp\left(j\left(\frac{N-1}{N}\right)\left(-r+i-\varepsilon_1\right)\right)\\
&= \frac{1}{N^2}\sum_{i\in\Gamma_1}\sum_{k=2}^{K}\sum_{m_k\in\Gamma_k} Z_{m_k}^{k}\,\frac{\sin\left(\pi\left(m_k-i+\varepsilon_k\right)\right)\sin\left(\pi\left(-r+i-\varepsilon_1\right)\right)}{\sin\left(\frac{\pi}{N}\left(m_k-i+\varepsilon_k\right)\right)\sin\left(\frac{\pi}{N}\left(-r+i-\varepsilon_1\right)\right)}\\
&\quad\cdot\exp\left(j\left(\frac{N-1}{N}\right)\left(m_k-r+\varepsilon_k-\varepsilon_1\right)\right).
\end{aligned}
\tag{23}
$$

As a result, the desired signal power, the inter-carrier interference power, and the multi-user interference power of the rth sub-carrier for the first cancellation are given in (24), (25) and (26), respectively.

$$
E\left[\left|\mathrm{D}(r,1)\right|^2\right] = \frac{E\left[\left|Z_r^1\right|^2\right]}{N^4}\cdot\left|\sum_{i\in\Gamma_1}\frac{\sin^2\left(\pi\left(r-i+\varepsilon_1\right)\right)}{\sin^2\left(\frac{\pi}{N}\left(r-i+\varepsilon_1\right)\right)}\right|^2, \tag{24}
$$

$$E\left[\left|\mathrm{I}_{\mathrm{ICI}}(r,1)\right|^2\right]=\frac{1}{N^4}\sum_{\substack{m_1\in\Gamma_1\\ m_1\neq r}}E\left[\left|Z_{m_1}^1\right|^2\right] \cdot\left|\sum_{i\in\Gamma_1}\frac{\sin\left(\pi\left(m_1-i+\varepsilon_1\right)\right)\sin\left(\pi\left(-r+i-\varepsilon_1\right)\right)}{\sin\left(\frac{\pi}{N}\left(m_1-i+\varepsilon_1\right)\right)\sin\left(\frac{\pi}{N}\left(-r+i-\varepsilon_1\right)\right)}\right|^2, \tag{25}$$

$$E\left[\left|\mathrm{I}_{\mathrm{MUI}}(r,1)\right|^2\right]=\frac{1}{N^4}\sum_{k=2}^{K}\sum_{m_k\in\Gamma_k}E\left[\left|Z_{m_k}^k\right|^2\right] \cdot\left|\sum_{i\in\Gamma_1}\frac{\sin\left(\pi\left(m_k-i+\varepsilon_k\right)\right)\sin\left(\pi\left(-r+i-\varepsilon_1\right)\right)}{\sin\left(\frac{\pi}{N}\left(m_k-i+\varepsilon_k\right)\right)\sin\left(\frac{\pi}{N}\left(-r+i-\varepsilon_1\right)\right)}\right|^2, \tag{26}$$

where $E[\cdot]$ denotes the average operation and $|(\cdot)|$ denotes the norm operation.

The signal to interference ratio (SIR) for the first cancellation of the *r*th sub-carrier can be represented as follows:

$$\mathrm{SIR}(r,1)=\frac{E\left[\left|\mathrm{D}(r,1)\right|^2\right]}{E\left[\left|\mathrm{I}_{\mathrm{ICI}}(r,1)\right|^2\right]+E\left[\left|\mathrm{I}_{\mathrm{MUI}}(r,1)\right|^2\right]}. \tag{27}$$

Using (24)-(26), the received SIR after the first cancellation is given by:

$$\mathrm{SIR}(r,1)=\frac{E\left[\left|Z_r^1\right|^2\right]\cdot\left|\sum_{i\in\Gamma_1}\frac{\sin^2\left(\pi\left(r-i+\varepsilon_1\right)\right)}{\sin^2\left(\frac{\pi}{N}\left(r-i+\varepsilon_1\right)\right)}\right|^2}{\sum_{k=1}^{K}\sum_{\substack{m_k\in\Gamma_k\\ m_k\neq r}}E\left[\left|Z_{m_k}^k\right|^2\right]\cdot\left|\sum_{i\in\Gamma_1}\frac{\sin\left(\pi\left(m_k-i+\varepsilon_k\right)\right)\sin\left(\pi\left(-r+i-\varepsilon_1\right)\right)}{\sin\left(\frac{\pi}{N}\left(m_k-i+\varepsilon_k\right)\right)\sin\left(\frac{\pi}{N}\left(-r+i-\varepsilon_1\right)\right)}\right|^2}. \tag{28}$$

After the detection stage, the interference regeneration stage generates the first user’s interference terms and cancels the first user’s interference from the received signal. For the second interference cancellation process, the multi-user interference power of the *r*th sub-carrier after circular convolution is given by:

$$E\left[\left|\mathrm{I}_{\mathrm{MUI}}(r,2)\right|^2\right]=\frac{1}{N^4}\sum_{k=3}^{K}\sum_{m_k\in\Gamma_k}E\left[\left|Z_{m_k}^k\right|^2\right]\cdot\left|\sum_{i\in\Gamma_1}\frac{\sin\left(\pi\left(m_k-i+\varepsilon_k\right)\right)\sin\left(\pi\left(-r+i-\varepsilon_1\right)\right)}{\sin\left(\frac{\pi}{N}\left(m_k-i+\varepsilon_k\right)\right)\sin\left(\frac{\pi}{N}\left(-r+i-\varepsilon_1\right)\right)}\right|^2$$
$$+\delta_1\cdot\frac{1}{N^4}\cdot\sum_{m_1\in\Gamma_1}E\left[\left|Z_{m_1}^1\right|^2\right]\cdot\left|\sum_{i\in\Gamma_2}\frac{\sin\left(\pi\left(m_1-i+\varepsilon_1\right)\right)\sin\left(\pi\left(-r+i-\varepsilon_2\right)\right)}{\sin\left(\frac{\pi}{N}\left(m_1-i+\varepsilon_1\right)\right)\sin\left(\frac{\pi}{N}\left(-r+i-\varepsilon_2\right)\right)}\right|^2, \tag{29}$$

where $r\in\Gamma_1$ and δ_1 is the residual cancellation error for the first user.

According to previous discussions, for the *p*th interference cancellation, the SIR of the *r*th sub-carrier after the circular convolution is given by:

$$\mathrm{SIR}(r,p)=\frac{E\left[\left|\mathrm{D}(r,p)\right|^2\right]}{E\left[\left|\mathrm{I}_{\mathrm{ICI}}(r,p)\right|^2\right]+E\left[\left|\mathrm{I}_{\mathrm{MUI}}(r,p)\right|^2\right]}, \tag{30}$$

where

$$E\left[\left|\mathrm{D}(r,p)\right|^2\right]=E\left[\left|Z_r^p\right|^2\right]\cdot\left|\sum_{i\in\Gamma_p}\frac{\sin^2\left(\pi\left(r-i+\varepsilon_p\right)\right)}{\sin^2\left(\frac{\pi}{N}\left(r-i+\varepsilon_p\right)\right)}\right|^2, \tag{31}$$

$$E\left[\left|\mathrm{I}_{\mathrm{ICI}}(r,p)\right|^2\right]=\sum_{\substack{m_p\in\Gamma_p\\ m_p\neq r}}E\left[\left|Z_{m_p}^p\right|^2\right]\cdot\left|\sum_{i\in\Gamma_p}\frac{\sin\left(\pi\left(m_p-i+\varepsilon_p\right)\right)\sin\left(\pi\left(-r+i-\varepsilon_p\right)\right)}{\sin\left(\frac{\pi}{N}\left(m_p-i+\varepsilon_p\right)\right)\sin\left(\frac{\pi}{N}\left(-r+i-\varepsilon_p\right)\right)}\right|^2, \tag{32}$$

$$E\left[\left|\mathrm{I}_{\mathrm{MUI}}(r,p)\right|^2\right]=\sum_{k=p+1}^{K}\sum_{m_k\in\Gamma_k}E\left[\left|Z_{m_k}^k\right|^2\right]\cdot\left|\sum_{i\in\Gamma_p}\frac{\sin\left(\pi\left(m_k-i+\varepsilon_k\right)\right)\sin\left(\pi\left(-r+i-\varepsilon_p\right)\right)}{\sin\left(\frac{\pi}{N}\left(m_k-i+\varepsilon_k\right)\right)\sin\left(\frac{\pi}{N}\left(-r+i-\varepsilon_p\right)\right)}\right|^2$$
$$+\sum_{q=1}^{p-1}\delta_q\cdot\sum_{m_q\in\Gamma_q}E\left[\left|Z_{m_q}^q\right|^2\right]\cdot\left|\sum_{i\in\Gamma_p}\frac{\sin\left(\pi\left(m_q-i+\varepsilon_q\right)\right)\sin\left(\pi\left(-r+i-\varepsilon_p\right)\right)}{\sin\left(\frac{\pi}{N}\left(m_q-i+\varepsilon_q\right)\right)\sin\left(\frac{\pi}{N}\left(-r+i-\varepsilon_p\right)\right)}\right|^2, \tag{33}$$

where $r\in\Gamma_p$ and δ_q is the residual cancellation error for user *q*.

B. ANALYSIS OF COMPLEXITY

In general, the complexity of circular convolution is higher than the DFT operation [5-6]. However, in the proposed scheme, complexity is significantly reduced since most of the elements to be convoluted are zeros. In (13), the number of nonzero terms of $\mathbf{Y}^p$ is equal to the number of the *p*th user's subcarriers. The number of nonzero terms of $\hat{\mathbf{I}}_{i,p}$ in (18) is equal to the number of the *p*th user's subcarriers subtract from the number of the total subcarriers. Furthermore, most elements in $\mathbf{C}^p$ and $\mathbf{C}^p{}'$ are quite small. In particular, the middle elements of $\mathbf{C}^p$ and $\mathbf{C}^p{}'$ are approximately zero. As a consequence, (14) can be written as

$$\mathbf{C}^p \approx \left[C_0^p, \cdots, C_{(m-1)/2}^p, 0, \cdots, 0, C_{N-(m-1)/2}^p, \cdots, C_{N-1}^p\right], \tag{34}$$

where m is the number of the reduced nonzero terms. Similarly, $\mathbf{C}^p{}'$ can be approximated as:

$$\mathbf{C}^{p}{}' \approx \left[C_0^{p}{}', \cdots, C_{(m-1)/2}^p{}', 0, \cdots, 0, C_{N-(m-1)/2}^p{}', \cdots, C_{N-1}^p{}'\right]. \tag{35}$$

From the above discussions, the circular convolutions in (13) and (18) can be written as a $N \times 1$ matrix multiplied by a $N \times N$ matrix. There are N/K nonzero elements in the $N \times 1$ matrix. In the $N \times N$ matrix, there are m nonzero elements in each row. As a result, the number of complex multiplications required to achieve the circular convolution in the proposed scheme is equal to Nm/K . As a comparison, the PIC architecture proposed by *Huang and Letaief* [6] requires $2Nm$ complex multiplications. As a result, the system complexity of the proposed SIC architecture is only 1/2*K*.

V. SIMULATION RESULTS

In this investigation, simulation experiments assume an OFDMA system with four users, $N = 256$ sub-carriers, and data is QPSK modulated. The subcarriers are uniformly interleaved across all the users and the CFO values are $\boldsymbol{\varepsilon} = [0.1 \ 0.08 \ 0.07 \ 0.05]$ for the four users. The system is assumed to have perfect power control. That is, the average received powers from all the users are the same at the BS. The channel model used here is a multi-path

Rayleigh fading channel, and all users' channels are assumed to be statistically independent and perfectly known at the BS. For each user, the coding scheme is a rate 1/2 convolutional code with a constraint length of 7, which is the same as the IEEE 802.16-2004 [1].

The average bit-error rates (BERs) are shown in Fig. 3 and Fig. 4 for uncoded and coded systems, respectively. Both figures reveal that, if the same m is adopted, the performance of the proposed SIC scheme with 2 or more iterations is better than that of the PIC proposed by *Huang and Letaief* [6].

Another factor that impacts system performance and complexity is the number of nonzero terms m in (20) and (21). Both system performance and complexity increase with m. To optimize m, simulation experiments are conducted for various m with 2 iterations. The results shown in Fig. 5 indicate that, when SNR$\leqq$15 dB, the difference between $m=3$ and $m=256$ is less than 1dB and the difference between $m=7$ and $m=256$ is negligible. However, for SNR$\geqq$20 dB, an increase in m significantly improves the system performance.

VI. Conclusions

In this paper, a successive interference cancellation structure is proposed for multi-user interference cancellation in the uplink of OFDMA systems. The proposed scheme adopts a circular convolution to compensate and regenerate the effects caused by CFOs. It is shown that the performance of the proposed SIC architecture with 2 iterations exceeds that of the PIC structure. However, the system complexity of the SIC is only 1/2K. This study also demonstrates that the system complexity can be significantly lessened if appropriate approximation is made by decreasing the number of nonzero terms m of $\mathbf{C}^p$ and $\mathbf{C}^{p}{}'$.

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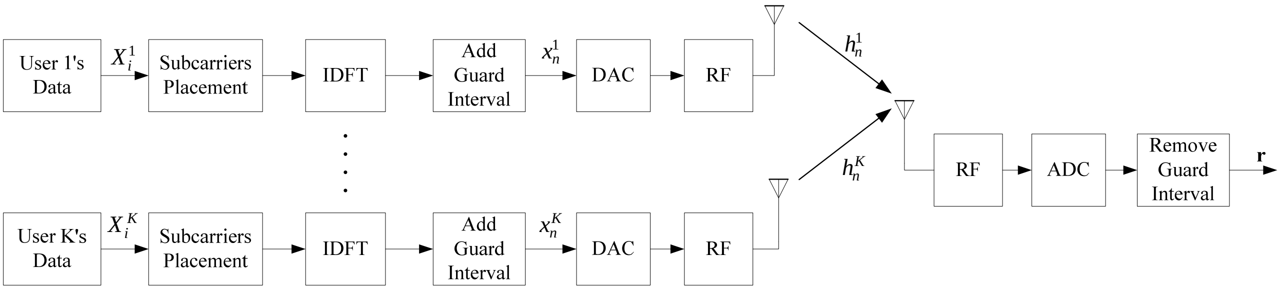

Fig. 1. Uplink transmission model for OFDMA systems

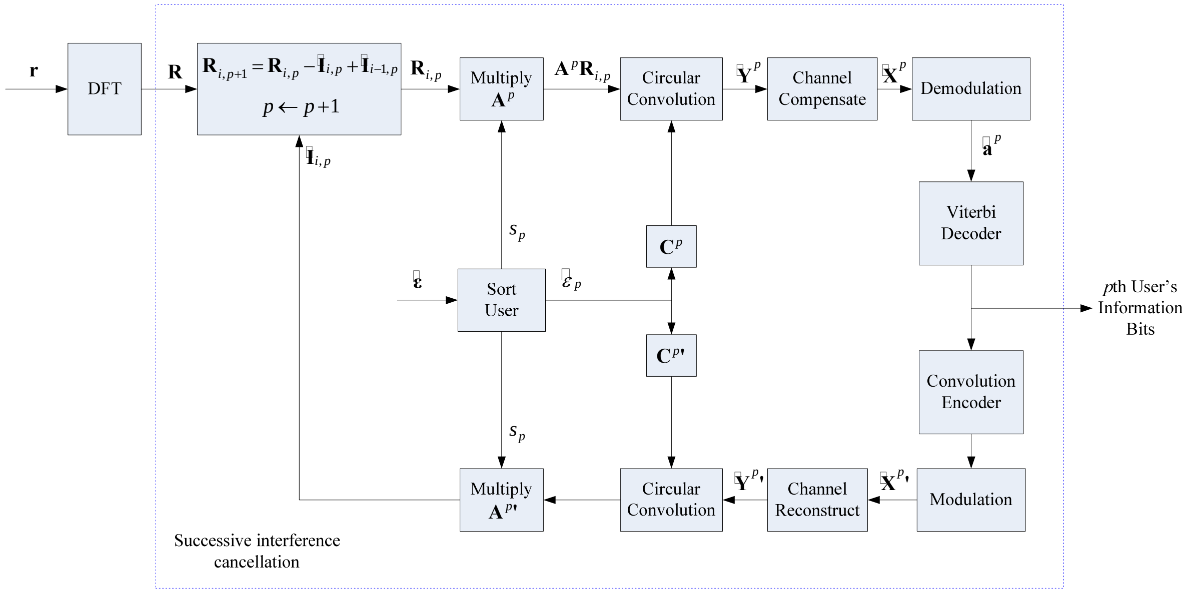

Fig. 2. Architecture for the proposed successive interference cancellation in the uplink of OFDMA systems

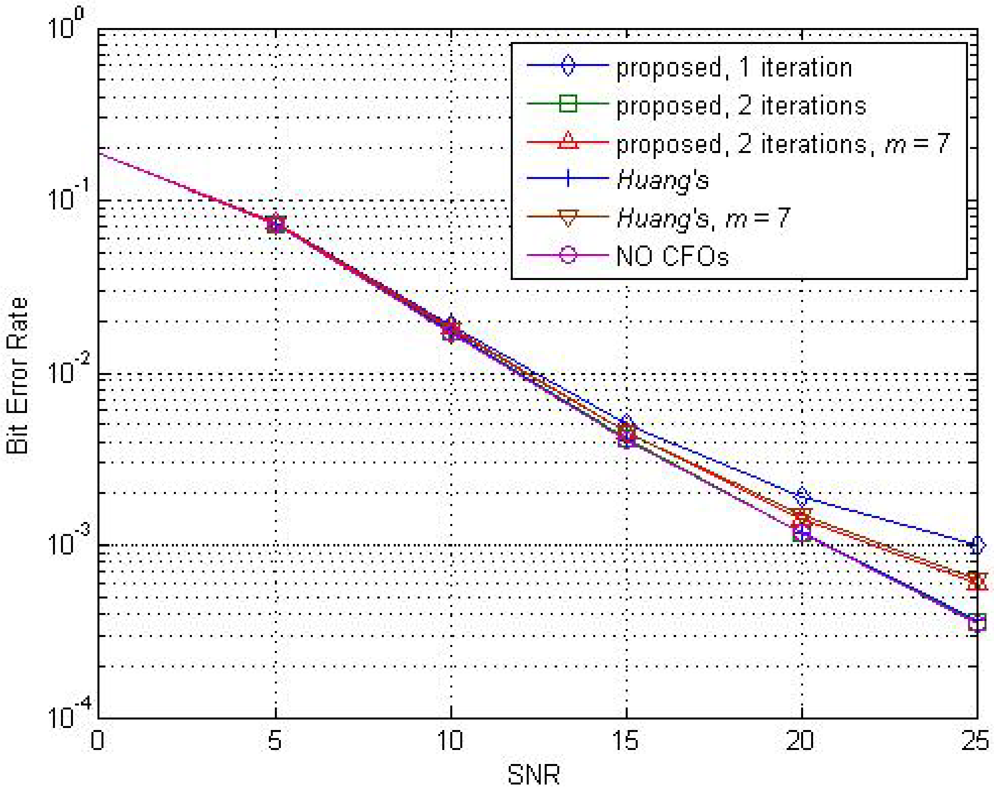

Fig. 3. BER performance of the SIC structure with uncoded QPSK modulation using perfect CFO and channel estimation

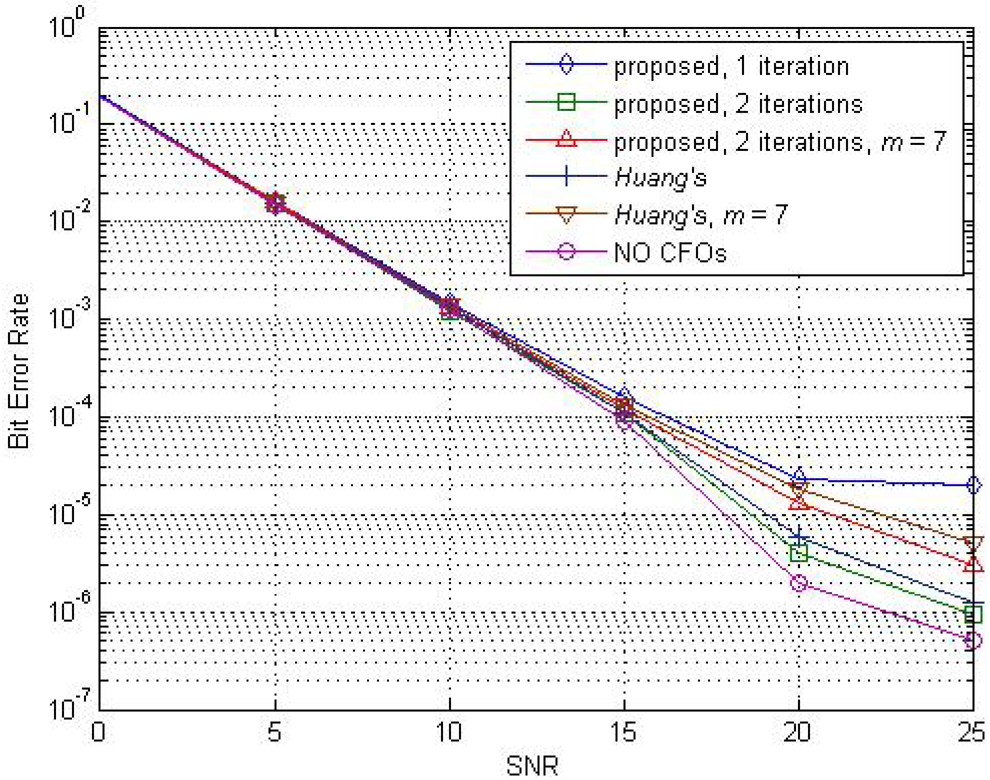

Fig. 4. BER performance of the SIC structure with coded QPSK modulation using perfect CFO and channel estimation

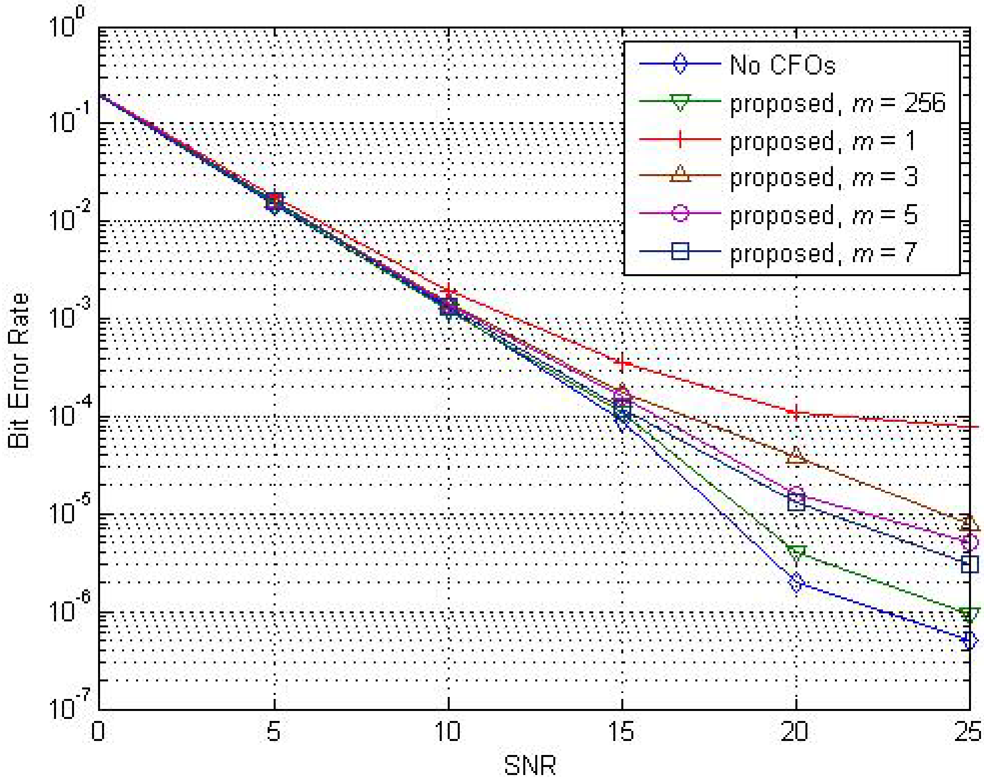

Fig. 5. BER performance of the SIC structure for various *m* with 2 iterations